Quantum spherical spin glass with random short-range interactions

Pedro Castro Menezes and Alba Theumann*
Instituto de Fisica, Universidade Federal do Rio Grande do Sul, Avenida Bento Gonçalves 9500, Caixa Postale 15051, 91501-970 Porto Alegre, Rio Grande do Sul, Brazil
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In the present paper we analyze the critical properties of a quantum spherical spin-glass model with short-range random interactions. Since the model allows for rigorous detailed calculations, we can show how the effective partition function calculated with the help of the replica method for the spin-glass fluctuating fields $Q_{\alpha\gamma}(\vec{k}\omega_1\omega_2)$ separates into a mean-field contribution for $Q_{\alpha\alpha}(0;\omega;-\omega)$ and a strictly short-range partition function for the fields $Q_{\alpha\neq\gamma}(\vec{k}\omega_1\omega_2)$. Here $\alpha,\gamma=1,\ldots,n$ are replica indices. The mean-field part W_{MF} coincides with previous results. The short-range part W_{SR} describes a phase transition in a Q^3-field theory, where the fluctuating fields depend on a space variable $\vec{r}$ and times τ_1 and τ_2. This we analyze using the renormalization group with dimensional regularization and minimal subtraction of dimensional poles. By generalizing standard field theory methods to our particular situation, we can identify the critical dimensionality as $d_c=5$ at very low temperatures due to the dimensionality shift $D_c=d_c+1=6$. We then perform an ϵ' expansion to order one loop to calculate the critical exponents by solving the renormalization-group equations.

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I. INTRODUCTION

After the formulation of the renormalization-group theory to explain the critical behavior and scaling properties of phase transitions, the natural question emerged of how this theory would apply to phase transitions in quantum systems. In these systems, time plays an essential role through the equations of motion of the operators even in equilibrium quantum statistical mechanics. Then a natural conjecture was that there would be a dimensional shift from the space dimension d to the effective $D=d+1$ and that the scaling behavior in the critical region would require the introduction of a new critical dynamical exponent z.[1,2] This is evident when the quantum-mechanical partition function is written as a functional integral in terms of fields that are functions of position and imaginary time τ variables, where $0\leq\tau\leq\beta$ and $\beta=\frac{1}{T}$ is the inverse temperature in units with $\hbar=k_B=1$, as phase transitions occur in infinite systems when the correlation length ξ becomes infinite at the critical temperature T_c. If $T_c>0$, the "length" β_c in the imaginary time direction is finite and the associated correlation length $\xi^z>\beta_c$, then the transition would be classical in d space dimensions. However, if quantum fluctuations drive the critical temperature to $T_c=0$, at this point the time length β_c is infinite and a transition with a dimensional shift $D=d+1$ is expected at a quantum critical point (QCP).[3]

Physical realizations of quantum phase transitions occur in strongly correlated systems[4] and other physical systems as described extensively in Ref. 5. A particular class of systems that present a quantum critical point are quantum spin glasses such as the insulating $LiHo_xY_{1-x}F_4$.[6] In this system a magnetic field applied perpendicular to the magnetic easy axis yields a splitting of the ground-state doublet. This splitting plays the role of a transverse field and the system is well represented by the Ising spin-glass model in a transverse field. The phase diagram[6] on the transverse field-temperature plane shows a reduction in the critical temperature with the increase in the transverse field up to a vanishing $T_c(\Gamma_c)=0$. The critical exponent γ_{eff} of the nonlinear susceptibility differs from its classical mean-field value corresponding to infinite-range interactions, suggesting the presence of short-range forces. Quantum phase transitions are also studied in the M-component spin-glass model in a transverse field[7] or by using the spin-glass model of M-component quantum rotors.[5,8] In the limit $M\to\infty$, the quantum rotor model[9] reduces to the quantum spherical model for a spin glass that has been studied before[10] in the mean-field limit of infinite-range interactions, following the classical spin-glass theory of Sherrington and Kirkpatrick (SK).[11] It was also shown that the effective action of the quantum spherical spin glass is invariant[12] under the Becchi-Rouet-Stora-Tyutin supersymmetry. Consequently the spin-glass order parameter vanishes, while replica symmetry (RS) is exact. The p-spin quantum spherical model was also studied by using the boson operator representation for the harmonic oscillator.[13,14] It was shown that the systems with $p=2$ and $p\geq3$ belong to different universality classes. For $p\geq3$ there is replica symmetry breaking (RSB) and the system belongs to the same universality class as the SK model, with a finite order parameter $Q\neq0$ below the critical temperature.

Since the formulation of SK spin-glass theory with infinite-range interactions, the natural question has been asked of how finite-range random interactions would modify the critical properties of spin glasses and which would be the critical exponent associated with them. To answer this question, renormalization-group calculations were performed above criticality[15] in an expansion in $\epsilon=6-d$ for short-range interactions, from where emerges $d_c=6$ as the critical dimensionality of the classical spin glass. Renormalization-group calculations for long-range interactions decaying as $r^{-(d+\sigma)}$ in the classical spin glass were also performed.[16,17] Below the critical temperature there is RSB and a nonvanishing order parameter that is in fact a matrix in replica space. Then a more difficult renormalization group in replica space should be performed, as discussed in detail in Ref. 18. Here also the critical dimensionality appears to be $d_c=6$, thus completing the description of the short-range classical spin glass below the critical temperature.

It is the purpose of this paper to analyze the critical properties of a quantum spherical spin glass with random short-range interactions by using renormalized perturbation theory with dimensional regularization and minimal subtraction of dimensional poles.[19] This task is far from trivial as the structure of the resulting field theory differs from standard theories. To start with, the quantum rotor model without disorder is covariant in space and time,[5] the frequency appearing just as a new momentum component to form a $(d+1)$-dimensional vector with modulus $k^2+\omega^2$, giving thus the value of the exponent $z=1$. But this is not the case in the disordered model. The disorder has no dynamical fluctuations and if average over disorder restores space translational invariance, this is not necessarily so in the time direction. Consequently, we are forced to consider an effective action in terms of a spin-glass field that depends on position $\vec{r}$ and on two imaginary times τ_1 and τ_2 (not on the two times' difference). This carries on the need to formulate in this paper new rules for the calculation of diagrams in the loop expansion.[19] We find accordingly that under renormalization the relation between space and imaginary time changes and as a result the exponent z differs from unity.

The quantum spherical model for a spin glass is the ideal testing ground for these ideas, as its formulation in terms of functional integrals allows for rigorous analytic calculations. As a difference with the infinite-range quantum spherical spin glass,[12] in the case of short-range interactions we have to use the replica method to derive the effective action, and this we do in Sec. II. In Sec. III we derive the renormalized perturbation theory to order one loop in an expansion in $\epsilon' = 6-D=5-d$. Then our critical space dimension is effectively $d_c=5$. We calculate the critical exponents by solving the renormalization-group equations in the critical region. We leave the discussions for Sec. IV. The detailed and far from trivial calculations are described in the Appendix to keep the natural flow of calculations in the paper.

II. MODEL

We consider a spin glass of quantum rotors with moment of inertia I in the spherical limit[9,10,12] with Hamiltonian

$$\mathcal{H}_{\rm SG} + \mu\sum_i S_i^2 = \frac{1}{2I}\sum_i P_i^2 - \frac{1}{2}\sum_{i,j} J_{ij}S_iS_j + \mu\sum_i S_i^2, \qquad (1)$$

where the spin variables at each site are continuous, $-\infty < S_i < \infty$, and we introduced the canonical momentum P_i with commutation rules:

$$[S_j, P_k] = i\delta_{j,k}. \qquad (2)$$

The sum in Eq. (1) runs over sites $i,j=1,\ldots,N$. The bond coupling J_{ij} in Eq. (1) is an independent random variable with the Gaussian distribution[15,16]

$$P(J_{ij}) = e^{-J_{ij}^2/2J^2V_{ij}}\sqrt{\frac{1}{2\pi J^2 V_{ij}}}, \qquad (3)$$

and $V_{ij}=V(\vec{R}_i-\vec{R}_j)$ is a short-range interaction with Fourier transform at low momentum k,

$$V(k) \approx 1-k^2. \qquad (4)$$

The chemical potential μ is a Lagrange multiplier that insures the mean spherical condition

$$-\frac{\partial\langle \ln \mathcal{W}\rangle}{\partial(\mu)} = \sum_i \int_0^\beta d\tau \langle S_i^2\rangle = \beta N, \qquad (5)$$

and $\beta=1/T$ is the inverse temperature. We work in units where the Boltzmann constant $k_B=\hbar=1$ and $\mathcal{W}$ is the quantum partition function,

$$\mathcal{W} = \text{Tr}\exp\left[-\beta\left(\mathcal{H}_{\rm SG} + \mu\sum_i S_i^2\right)\right], \qquad (6)$$

which can be expressed as a functional integral,[12,20,21]

$$\mathcal{W} = \int \prod_i \mathcal{D}S_i \exp(-\mathcal{A}_O - \mathcal{A}_{\rm SG}), \qquad (7)$$

where the noninteracting action $\mathcal{A}_O$ is given by

$$\mathcal{A}_O = \int_0^\beta d\tau \sum_i \left[\frac{I}{2}\left(\frac{\partial S_i}{\partial\tau}\right)^2 + \mu S_i^2(\tau)\right] \qquad (8)$$

and the interacting part by

$$\mathcal{A}_{\rm SG} = \frac{1}{2}\sum_{i,j} J_{ij}\int_0^\beta d\tau S_i(\tau)S_j(\tau). \qquad (9)$$

The free energy may be calculated with the replica method,

$$F = -\frac{1}{\beta N}\lim_{n\to 0}\frac{W_n-1}{n}, \qquad (10)$$

where $\langle \mathcal{W}^n\rangle_{\rm ca}=W_n$ is the partition functional for n-identical replicas, configurationally averaged over the probability distribution of J_{ij} in Eq. (3). It is shown in the Appendix that W_n may be expressed as a functional over fluctuating spin-glass fields $Q_{\alpha\gamma}(\vec{k},\omega,\omega')$, where $\omega=\frac{2\pi m}{\beta}$ is a discrete Matsubara frequency for finite temperature and $\alpha,\gamma=1,\ldots,n$ are replica indices. The result obtained in Eq. (A10) of the Appendix is that the partition functional separates into two parts,

$$W_n = W_{\rm MF}W_{\rm SR}, \qquad (11)$$

where $W_{\rm MF}$ in Eq. (A11) is the mean-field functional for the fields $Q_{\alpha\alpha}(0,\omega,-\omega)$ obtained in Ref. 12, which determines the critical temperature $T_c(I)$ shown in the phase diagram in Fig. 1. On the other hand $W_{\rm SR}$ depends on the spin-glass fluctuations $Q_{\alpha\neq\gamma}(\vec{k},\omega,\omega')$ for short-range interactions and determines the critical behavior. We remark that these fields depend naturally on two independent times (frequencies) and not on the difference between the two times because the disorder is not time correlated and it restores translational invariance in space but not in time.[8] We obtain from Eq. (A13)

$$W_{\rm SR} = \int \prod_{\alpha\neq\gamma} \mathcal{D}Q_{\alpha\gamma}(\vec{k},\omega,\omega')\exp(-A_{\rm SR}\{Q\}), \qquad (12)$$

where $\alpha,\gamma=1,\ldots,n$ are replica indices and

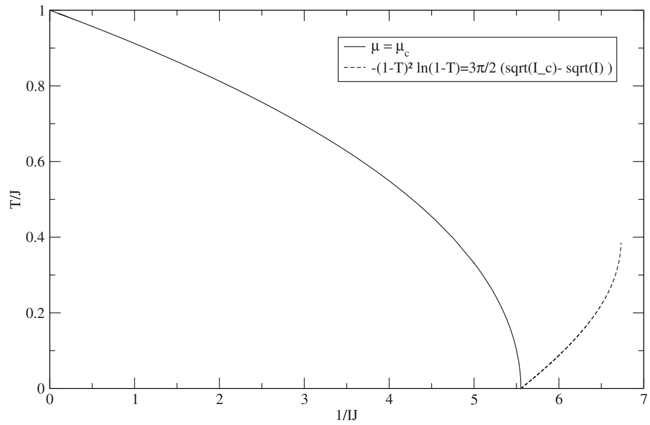

FIG. 1. Phase diagram in the T vs $1/I$ plane. The critical line $T_c(1/I)$ (full) separates the classical paramagnetic phase (top) from the spin-glass phase (bottom). The estimated line (dots) separates the classical paramagnetic region (top) from the quantum paramagnetic region (bottom).

$$A_{\rm SR}\{Q\} = \sum_{\alpha\neq\gamma}\sum_{\omega_1\omega_2}\int d\vec{k}\left[\frac{\mu-\mu_c}{\mu_c}+k^2+s^2(\omega_1^2+\omega_2^2)\right]\times Q_{\alpha\gamma}(\vec{k},\omega_1,\omega_2)Q_{\alpha\gamma}(-\vec{k},-\omega_1,-\omega_2) + \frac{\lambda}{3!}\sum_{\alpha\neq\gamma\neq\delta}\sum_{\omega_1\omega_2\omega_3}\int d\vec{k}_1 d\vec{k}_2 Q_{\alpha\gamma}(\vec{k}_1,\omega_1,\omega_2)\times Q_{\gamma\delta}(\vec{k}_2,-\omega_2,\omega_3)Q_{\delta\alpha}(-\vec{k}_1-\vec{k}_2,-\omega_3,-\omega_1). \tag{13}$$

Having in mind a renormalization-group calculation, the frequency term in the noninteracting inverse propagator is affected by the coefficient s, as it will turn out that momentum and frequency renormalize differently and they cannot be kept both equal to unity. The infinite-volume limit was taken in Eq. (13), but for the moment the temperature is kept finite and the sums are over discrete Matsubara frequencies. In all the following work, it is implicit that $Q_{\alpha\gamma}$ means $Q_{\alpha\neq\gamma}$, while $Q_\alpha(\omega)$ means $Q_{\alpha\alpha}(0,\omega,-\omega)$. There is also a coupling $Q_{\alpha\alpha}(\vec{q}\neq 0)Q_{\alpha\neq\gamma}Q_{\gamma\neq\alpha}$ that generates an internal propagator line $G_{\alpha\alpha}(\vec{q}\neq 0)$ in the loops in Fig. 2, where the internal line has $\gamma=\alpha$. The contribution of these diagrams is given by a sum $\Sigma_{\vec{q}\neq 0}$, which presents a smaller degree of infrared divergence at the critical theory, when $\mu=\mu_c$, and can be neglected.

We now proceed with the renormalization-group calculation using dimensional regularization and minimal subtraction of dimensional poles,[19] to one loop order. In Eq. (13) we kept only the terms $O(Q^3)$ because the terms $O(Q^4)$ would be irrelevant close to the critical dimensionality of a Q^3 theory, as there is no change in the sign of λ for the Gaussian probability distribution of the random bonds.[15] To analyze the value of the critical dimensionality, we consider separately the case of finite temperature from that of $T=0$. In both cases the vertex functions that present divergences needing renormalization are the inverse propagator $\Gamma^{(2)}$, the three-point vertex function $\Gamma^{(3)}$, and the two-point vertex function with one insertion $\Gamma^{(2,1)}$.[19] To one loop order they are given by the diagrams in Fig. 2. At this point it is important to distinguish between the system temperature T and the critical parameter $t=\frac{\mu-\mu_c}{\mu_c}$ that measures the approach to criticality.

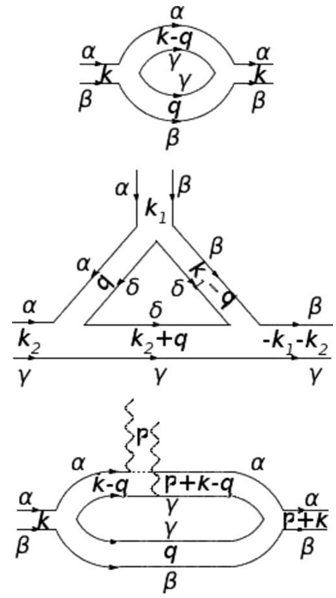

FIG. 2. Diagrammatic representation of the vertex functions. A double line represents a propagator with two replica indices α,γ, momentum $\vec{k}$, and two frequencies ω_1,ω_2. Top: $\Gamma^{(2)}$; middle: $\Gamma^{(3)}$; bottom: $\Gamma^{(2,1)}$.

We start by analyzing the transition at finite temperature T. The action in Eq. (13) must be dimensionless. Then dimensional analysis tells us that for Λ an inverse length

$$[k]=\Lambda,\quad [Q]=\Lambda^{-d/2-1},\quad [\lambda]=\Lambda^{3-d/2} \tag{14}$$

and the critical dimensionality is $d_c=6$, as corresponds to a classical system. The vertex functions calculated with the usual rules in φ^3-field theory[19,22] are

$$\Gamma^{(2)}(\vec{k},\omega_1,\omega_2)=\Gamma^{(0)}(\vec{k},\omega_1,\omega_2)-(n-2)\frac{1}{2}\lambda^2\times\sum_{\omega}\int d\vec{p}G_0(\vec{p},\omega,\omega_1)G_0(\vec{k}-\vec{p},\omega_2,-\omega), \tag{15}$$

where

$$\Gamma^{(0)}(\vec{k},\omega_1,\omega_2)=t+k^2+s^2(\omega_1^2+\omega_2^2)=G_0^{-1}(\vec{k},\omega_1,\omega_2) \tag{16}$$

and

$$\Gamma^{(3)}(\vec{k}_1,\vec{k}_2,\omega_1,\omega_2,\omega_3) = \lambda+(n-3)\lambda^3\sum_{\omega}\int d\vec{p}G_0(\vec{p},\omega_1,\omega)\times G_0(\vec{k}_1+\vec{p},-\omega,\omega_2)G_0(\vec{k}_1+\vec{k}_2+\vec{p},-\omega,\omega_3). \tag{17}$$

The theory will be renormalized at the critical point $t=0$. To get away from the critical point, we should consider a perturbation expansion in t by means of the insertion[19]

$$\Delta A = \frac{1}{2!}\sum_{\gamma,\nu}\sum_{\omega_1\omega_2}\int d\vec{q}t(\vec{q})\int d\vec{p}Q_{\gamma\nu}(\vec{p},\omega_1,\omega_2) \times Q_{\nu\gamma}(\vec{q}-\vec{p},-\omega_2,-\omega_1), \qquad (18)$$

which leads to a third singular vertex function $\Gamma^{(2,1)}$ with two external legs and one insertion shown in Fig. 2 (bottom),

$$\Gamma^{(2,1)}(\vec{k},\vec{q},\omega_1,\omega_2) = 1 + (n-2)\lambda^2\sum_{\omega}\int d\vec{p}G_0(\vec{p},\omega,\omega_1) \times G_0(\vec{q}-\vec{p},-\omega_1,-\omega)G_0(\vec{k}+\vec{p},\omega,\omega_2). \qquad (19)$$

At finite temperature T and critical $t=0$, the sums over Matsubara frequencies have only one singular term with $\omega=0$ and the vertex functions are singular when $\omega_i=0$. Then we recover the transition for classical spin glasses described by an expansion in $\epsilon=6-d$.[15]

A different scenario emerges when T is near zero. For sufficiently low T the frequency sums may be replaced by integrals,

$$\sum_{\omega} \rightarrow \beta\int_{-\infty}^{\infty} d\omega, \qquad (20)$$

and now all the frequencies contribute to the renormalization process and the vertex functions in Eqs. (15), (17), and (19) will be singular at a new effective dimension $D_c=d_c+1=6$, the new critical space dimensionality becoming $d_c=5$, as predicted.[1–3,5]

III. RESULTS

In the following we present results for the critical properties in an expansion in $\epsilon'=5-d$, to one loop order. The new features that emerge from the calculation are that the frequencies renormalize differently from the momenta and then the exponent z differs from unity and from the exponent η, depending also on the dimensionality through the ϵ' expansion. The integrals over momentum and frequency in Eqs. (15), (17), and (19) are calculated in the Appendix at a space dimensionality d, when they converge[19,22] and the singularities appear as dimensional poles in ϵ'. We obtain for the singular parts, to leading order in the coupling constant,

$$\Gamma^{(2)}_{\alpha\gamma}(\vec{k},\omega_1,\omega_2) = k^2 + s^2(\omega_1^2+\omega_2^2) + (n-2)\frac{1}{6s\epsilon'}u_0^2[k^2+3s^2(\omega_1^2+\omega_2^2)] = \left[1+(n-2)\frac{1}{6s\epsilon'}u_0^2\right] \times\left[k^2+s^2(\omega_1^2+\omega_2^2)\left(1+\frac{n-2}{3\epsilon'}u_0^2\right)\right], \qquad (21)$$

$$\Gamma^{(3)}_{\alpha\gamma\delta} = u_0\kappa^{\epsilon'/2}\left[1+(n-3)u_0^2\frac{1}{s\epsilon'}\right], \qquad (22)$$

$$\Gamma^{(2,1)}_{\alpha\gamma} = 1+(n-2)u_0^2\frac{1}{s\epsilon'}, \qquad (23)$$

where we introduced the bare dimensionless coupling u_0 through

$$\lambda^2\beta S_{d+1} = u_0^2\kappa^{\epsilon'} \qquad (24)$$

and S_{d+1} is the surface of the unit sphere in $d+1$ dimensions. In $\Gamma^{(3)}$ and $\Gamma^{(2,1)}$ the external momenta and frequencies were taken at the symmetry point $k_1^2=k_2^2=-2\vec{k}_1\cdot\vec{k}_2=\omega_i^2=\kappa^2$, where κ is the scale parameter.[19] In order to cancel the dimensional poles, we must introduce a renormalized dimensionless coupling u and renormalized vertex functions by means of renormalization of the field $Q_{\alpha\gamma}$ and of the insertion $Q^2_{\alpha\gamma}$ through the functions Z_Q and Z_{Q^2}. The correction to the frequency term ω_i^2 in $\Gamma^{(2)}$ in Eq. (21) is different from the contribution to k^2. Then besides the field renormalization Z_Q that keeps the coefficient of k^2 equal to unity, it is necessary to renormalize also the frequency coefficient $s(u)$. All together we obtain

$$\Gamma^{(2)}_R(u) = Z_Q(u)\Gamma^{(2)}(u_0,s),$$

$$\Gamma^{(3)}_R(u) = Z_Q^{3/2}(u)\Gamma^{(3)}(u_0),$$

$$\Gamma^{(2,1)}_R(u) = \bar{Z}_{Q^2}(u)\Gamma^{(2,1)}(u_0), \qquad (25)$$

where in the interesting limit $n=0$

$$u_0 = u\left(1+\frac{5}{2\epsilon'}u^2\right), \qquad (26a)$$

$$Z_Q(u) = 1+\frac{1}{3\epsilon'}u^2, \qquad (26b)$$

$$\bar{Z}_{Q^2}(u) = 1+\frac{2}{\epsilon'}u^2, \qquad (26c)$$

$$s^2 = 1+\frac{2}{3\epsilon'}u^2. \qquad (26d)$$

From Eq. (26a) we calculate the β function

$$\beta(u) = \kappa\left.\frac{\partial u}{\partial\kappa}\right|_{\lambda} = -\frac{\epsilon'}{2}u\left(1-\frac{5}{\epsilon'}u^2\right), \qquad (27)$$

which vanishes at the trivial fixed points $u^*=0$ (stable for $\epsilon'<0$) and $u^{*2}=\frac{1}{5}\epsilon'$ (stable for $\epsilon'>0$). To obtain the critical exponents, we have to solve the renormalization-group equations[19] for the vertex function $\Gamma^{(2)}_R(\vec{k},s\omega_i,t,u,\kappa)$ near criticality, where $t=\frac{\mu-\mu_c}{\mu_c}$. Now we have to take into account also the dependence of s on κ through the coupling u, so setting $y_i=s\omega_i$, $i=1,2$, we obtain the renormalization-group equation at the fixed point $\beta(u^*)=0$,

$$\left[\kappa\frac{\partial}{\partial\kappa}+\gamma_s^*\sum_i y_i\frac{\partial}{\partial y_i}-\theta t\frac{\partial}{\partial t}-\eta\right]\Gamma_R^{(2)}(\vec{k},y_i,t,\kappa)=0, \quad (28)$$

where

$$\eta=\left[\kappa\frac{\partial}{\partial\kappa}\ln Z_Q\right]_{u=u^*},$$

$$\theta=\left[\kappa\frac{\partial}{\partial\kappa}\ln \bar{Z}_{Q^2}\right]_{u=u^*}-\eta,$$

$$\gamma_s^*=\left[\kappa\frac{\partial}{\partial\kappa}\ln s\right]_{u=u^*}. \quad (29)$$

The solution for $\Gamma_R^{(2)}$ has the scaling form

$$\Gamma_R^{(2)}(\vec{k},y_i,t,\kappa)=\kappa^{\eta}\Phi[\vec{k};y_i t\kappa^{\theta-\gamma_s^*}], \quad (30)$$

where Φ is a function of the joint variable $y_i t\kappa^{\theta-\gamma_s^*}$ and dimensional analysis tells us that for ρ an inverse length:

$$\Gamma_R^{(2)}(\vec{k},y_i,t,\kappa)=\rho^2\Gamma_R^{(2)}\left(\frac{\vec{k}}{\rho},\frac{y_i}{\rho},\frac{t}{\rho^2},\frac{\kappa}{\rho}\right)$$

$$=\rho^2\left[\frac{\kappa}{\rho}\right]^{\eta}\Phi\left[\frac{\vec{k}}{\rho};\frac{y_i t\kappa^{\theta-\gamma_s^*}}{\rho^{3+\theta-\gamma_s^*}}\right]. \quad (31)$$

If we choose[19]

$$\rho=\kappa\left(\frac{t}{\kappa^2}\right)^{1/(2+\theta)}, \quad (32)$$

we obtain

$$\Gamma_R^{(2)}(\vec{k},y_i,t,\kappa)=\kappa^2\left(\frac{t}{\kappa^2}\right)^{\nu(2-\eta)}\Phi\left[\frac{\vec{k}}{\kappa}\left(\frac{t}{\kappa^2}\right)^{-\nu};\frac{y_i}{\kappa}\left(\frac{t}{\kappa^2}\right)^{-\nu z}\right], \quad (33)$$

from where we identify the space correlation length exponent

$$\xi=\left(\frac{t}{\kappa^2}\right)^{-\nu}, \quad \nu^{-1}=2+\theta \quad (34)$$

and the time correlation length exponent

$$\xi_t=\left(\frac{t}{\kappa^2}\right)^{-\nu z}=\xi^z, \quad z=1-\gamma_s^*. \quad (35)$$

From Eqs. (26), (29), (34), and (35) we obtain the results for the critical exponents, at the nontrivial fixed point

$$\eta=-\frac{1}{15}\epsilon', \quad \nu=\frac{1}{2}+\frac{1}{12}\epsilon', \quad z=1+\frac{1}{15}\epsilon'. \quad (36)$$

Scaling theory gives for the static spin-glass susceptibility $\chi^{-1}=\Gamma_R^{(2)}(0,0,t)\approx t^{\gamma}$, with $\gamma=\nu(2-\eta)$, from Eq. (33).

IV. CONCLUSIONS

In the present paper we analyze the critical properties of a quantum spherical spin-glass model with short-range random interactions. Since the model allows for rigorous detailed calculations, we can show how the effective partition function calculated with the help of the replica method for the spin-glass fluctuating fields $Q_{\alpha\gamma}(\vec{k},\omega_1,\omega_2)$ separates into a mean-field contribution for $Q_{\alpha\alpha}(0,\omega,-\omega)$ and a strictly short-range partition function for the fields $Q_{\alpha\neq\gamma}(\vec{k},\omega_1,\omega_2)$. Here $\alpha,\gamma=1,\ldots,n$ are replica indices. The mean-field part W_{MF} coincides with previous results[12] and a saddle-point calculation provides the phase diagram in Fig. 1, as discussed in the Appendix. This phase diagram agrees with the measurements in Ref. 6 as the inverse of the rotor's moment of inertia measures the strength of the quantum fluctuations and plays the role of the transverse field in the $\mathrm{LiHo}_x\mathrm{Y}_{1-x}\mathrm{F}_4$ alloys. We stress that $Q_{\alpha\alpha}(0,\omega,-\omega)$ is not an order parameter, as it does not vanish above the transition temperature, and the order parameter in the quantum spherical infinite-range spin glass identically vanishes.[12] The short-range part W_{SR} describes a phase transition in a Q^3-field theory, where the fluctuating fields depend on a space variable $\vec{r}$ and times τ_1 and τ_2. This we analyze using the renormalization group with dimensional regularization and minimal subtraction of dimensional poles.[19] By generalizing the method in Ref. 19 to our particular situation, we can identify the critical dimensionality as $d_c=5$ at very low temperatures due to the dimensionality shift $D_c=d_c+1=6$. We then perform an ϵ' expansion to order one loop to calculate the critical exponents by solving the renormalization-group equations; they are listed in Eq. (36).

A general Landau theory of quantum spin glasses of M-component rotors was presented in Ref. 8. Based on the general properties of symmetry and invariance, the authors presented an effective functional for spin-glass Q fields, and at some points we make contact with their results. Our fields, as theirs, are bilocal in time, but our result for the effective functional is simpler and more tractable by standard field theory methods. It is well known[23] than the classical nonrandom spherical model is equivalent to the $M\rightarrow\infty$ limit of the M-vector model. The same equivalence holds between the infinite-range spherical spin glass and the infinite-range M-vector spin glass in the classical case[24,25] and in the quantum case.[9,12] A particular feature of the infinite-range spherical spin glass is that it can be solved exactly without need of the replica method[24] because annealing is exact in this model due to the internal Becchi-Rouet-Stora-Tyutin (BRST) supersymmetry.[12] As a consequence Ward identities tell us that the order parameter identically vanishes. In the case of the short-range quantum spherical spin glass considered here, we showed that replicas are needed and that the partition functional separates exactly into a mean-field part for the replica diagonal $Q_{\alpha\alpha}(k=o,\omega,-\omega)$ and a short-range part for the fluctuating $Q_{\alpha\neq\beta}(k,\omega_1,\omega_2)$ in Eq. (11). On the other hand in the spin glass of M-component quantum rotors with short-range disorder considered in Ref. 8, the replica diagonal $Q_{\alpha\alpha}(\omega)$ is considered as an order parameter and a Landau functional is constructed for fluctuations diagonal in replica space around it. This leads to a theory where the time derivatives and the critical mass appear in the *linear* term, in place of the quadratic one, in the effective action. As a consequence of having different interactions, the renormalization-group equations and critical exponents turn out to be M independent, and the critical dimensionality obtained in Ref. 8 also differs from ours. We conclude this is due to the fact that in the case of short-range disorder considered here, the quantum spherical spin-glass model belongs to a different universality class from the M-component quantum rotor model in Ref. 8. Ultimately we compare our results with Monte Carlo simulations in $(d+1)$ dimensions for $d=3$ (Ref. 26) and for $d=2$ (Ref. 27) for the Ising spin glass in a transverse field. The scaling form for the inverse spin-glass susceptibility χ_{SG}^{-1} in Eq. (33) agrees with Ref. 26, which yields for the susceptibility exponent the scaling result $\gamma=\nu(2-\eta)$. Calculations in $(2+1)$ dimensions[27] confirm this result for γ_{SG}. We also obtain $1<z<2$, as in the above-mentioned references.[26,27] The comparison cannot go further as we are dealing with a system with high values of $M\to\infty$ and high dimensionality $D=5+1$. Quantum Heisenberg spin-glass systems were also studied recently with numerical methods.[28]

We may ask how our results would apply to the quantum p-spin spherical model theories in Refs. 13 and 14 in the case of short-range disorder. In these theories the action depends on the first time derivative of the fields. Then the inverse propagators would have a linear dependence on frequency (and not quadratic, as it is the case here), so the results of a renormalization-group (RG) calculation remain open.

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APPENDIX

1. Effective functional

We derive here the functional W_n in Eq. (9). We obtain by replicating $\mathcal{W}$ in Eq. (7) and averaging over $P(J_{ij})$ in Eq. (3) the following:

$$W_n = \int \prod_{i\alpha} \mathcal{D}S_{i\alpha} \exp(-\mathcal{A}_0 - A_{SG}), \tag{A1}$$

where $\alpha=1,2,\ldots,n$ is the replica index and the free action $\mathcal{A}_0$ is given by

$$\mathcal{A}_0 = \int_0^\beta d\tau \sum_i \sum_\alpha \left[\frac{I}{2}\left(\frac{\partial S_{i\alpha}}{\partial\tau}\right)^2 + \mu S_{i\alpha}^2(\tau)\right], \tag{A2}$$

while the interacting part is

$$\mathcal{A}_{SG} = \frac{J^2}{4}\sum_{i,j}\sum_{\alpha\alpha'} V_{ij}\int_0^\beta d\tau \int_0^\beta d\tau' S_{i\alpha}(\tau)S_{j\alpha}(\tau)S_{i\alpha'}(\tau')S_{j\alpha'}(\tau'). \tag{A3}$$

We introduce the spin-glass fields $Q_{i\alpha\gamma}(\tau,\tau')$ by splitting the quartic term by means of a Stratonovich-Hubbard transformation and we obtain

$$W_n = \int \prod_i \prod_{\alpha\gamma} \mathcal{D}Q_{i\alpha\gamma}(\tau,\tau') \exp\left[-\frac{J^2}{4}\sum_{\alpha\gamma}\int_0^\beta\int_0^\beta d\tau d\tau' \times \sum_{i,j} Q_{i\alpha\gamma}(\tau,\tau')V_{i,j}^{-1}Q_{j\gamma\alpha}(\tau,\tau')\right]\exp[N\Lambda], \tag{A4}$$

where

$$\exp[N\Lambda] = \int \prod_{i,\alpha}\mathcal{D}S_{i\alpha}(\tau)\exp\left[-\mathcal{A}_0 - \frac{J^2}{2}\sum_{\alpha\gamma}\int_0^\beta\int_0^\beta d\tau d\tau' \times \sum_j Q_{j\alpha\gamma}(\tau,\tau')S_{j\alpha}(\tau)S_{j\gamma}(\tau')\right]. \tag{A5}$$

In Eq. (A5) appear the fields $Q_{j\alpha\gamma}(\tau,\tau')$ depending on two independent times τ,τ' and not on the time difference. We define the space and time Fourier transform as

$$S_\alpha(\vec{k}\omega) = \frac{1}{\beta\sqrt{N}}\int_0^\beta d\tau \sum_j S_{j\alpha}(\tau)\exp - i[\vec{k}\cdot\vec{R}_j + \omega\tau], \tag{A6}$$

$$Q_{\alpha\gamma}(\vec{k}\omega\omega') = \frac{1}{\beta^2 N}\int_0^\beta d\tau \int_0^\beta d\tau' \sum_j Q_{j\alpha\gamma}(\tau\tau') \times \exp - i[\vec{k}\cdot\vec{R}_j + \omega\tau + \omega'\tau'], \tag{A7}$$

where $\omega=\frac{2\pi m}{\beta}$ are bosonic Matsubara frequencies and we obtain from Eqs. (A4)–(A7)

$$W_n = \int \prod_{\vec{k}\omega_1\omega_2}\prod_{\alpha\gamma} dQ_{\alpha\gamma}(\vec{k}\omega_1\omega_2) \times \exp\left[-\frac{(\beta J)^2}{4}\sum_{\alpha\gamma}\sum_{\vec{k}\omega_1\omega_2} Q_{\alpha\gamma}(\vec{k}\omega_1\omega_2)V(\vec{k})^{-1} \times Q_{\alpha\gamma}(-\vec{k},-\omega_1,-\omega_2)\right]\exp[N\Lambda], \tag{A8}$$

where

$$\exp[N\Lambda] = \int \prod_\alpha \prod_{\vec{k}\omega} dS_\alpha(\vec{k}\omega)\exp\left[-\sum_\alpha\sum_{\vec{k}\omega}\left(\frac{\beta I\omega^2}{2} + \mu\beta\right) \times S_\alpha(\vec{k}\omega)S_\alpha(-\vec{k}-\omega)\right] \times \exp\left[\frac{(\beta J)^2}{2}\sum_{\alpha\gamma}\sum_{\vec{k}\vec{q}}\sum_{\omega\omega'} Q_{\alpha\gamma}(\vec{q}\omega\omega') \times S_\alpha(\vec{k}\omega)S_\gamma(\vec{k}-\vec{q},\omega')\right], \tag{A9}$$

and for short-range forces $V(k)^{-1}=1+k^2$. The next step is to separate the term with $Q_{\alpha\alpha}(0,\omega,-\omega)$ in Eq. (A9) that can be introduced into the free action for $S_\alpha(\vec{k}\omega)$, with the result

$$W_n = W_{\mathrm{MF}} W_{\mathrm{SR}}, \tag{A10}$$

where W_{MF} is the mean-field partition functional for the $Q_\alpha(\omega)=Q_{\alpha\alpha}(0\omega-\omega)$ mode,

$$W_{\mathrm{MF}} = \int \prod_{\omega\alpha} dQ_\alpha(\omega)\exp - \frac{N}{2}\sum_{\alpha\omega}\left\{\frac{(\beta J)^2}{2}Q_\alpha^2(\omega) + \ln[\beta I\omega^2 + 2\beta\mu - (\beta J)^2 Q_\alpha(\omega)]\right\} \tag{A11}$$

and W_{SR} is the partition functional for the fluctuations $Q_{\alpha\neq\gamma}$,

$$W_{\mathrm{SR}} = \int \prod_{\alpha\neq\gamma}\prod_{\vec{k}\omega} dQ_{\alpha\neq\gamma}(\vec{k}\omega_1\omega_2)\exp - [\mathcal{A}_{\mathrm{free}} + \mathcal{A}_{\mathrm{int}}], \tag{A12}$$

$$\mathcal{A}_{\mathrm{free}} = N\sum_{\vec{k}\omega_1\omega_2}\sum_{\alpha\gamma} Q_{\alpha\gamma}(\vec{k}\omega_1\omega_2)Q_{\gamma\alpha}(-\vec{k},-\omega_1,-\omega_2) \times(\beta J)^2\Gamma_0(\vec{k}\omega_1\omega_2), \tag{A13}$$

$$\mathcal{A}_{\mathrm{int}} = \frac{(\beta J)^6}{3!}N\sum_{\vec{k}_1\vec{k}_2}\sum_{\alpha\omega_1}\sum_{\gamma\omega_2}\sum_{\delta\omega_3} Q_{\alpha\gamma}(\vec{k}_1\omega_1\omega_2) \times Q_{\gamma\delta}(\vec{k}_2,-\omega_2,\omega_3)Q_{\delta\alpha}(-\vec{k}_1-\vec{k}_2,-\omega_3,-\omega_1)\prod_i g_0(\omega_i), \tag{A14}$$

where

$$\Gamma_0(\vec{k}\omega_1\omega_2) = 1 + k^2 - (\beta J)^2 g_0(\omega_1)g_0(\omega_2). \tag{A15}$$

The function $g_0(\omega)$ in Eq. (A14) is the momentum independent, noninteracting two-point function for the field $S_\alpha(\omega)$,

$$g_0(\omega) = \frac{2}{\beta I\omega^2 + 2\beta\mu - (\beta J)^2 Q_\alpha(\omega)}. \tag{A16}$$

The variables $Q_{\alpha\alpha}(\vec{q}\neq 0,\omega_1\omega_2)$ are not critical and are not coupled to the spin-glass field, so we ignore them.

2. Mean-field solution

At the saddle point of W_{MF} in Eq. (A11), we obtain

$$2Q_\alpha(\omega) = g_0(\omega). \tag{A17}$$

The mean spherical condition in Eq. (5) reduces to

$$-\frac{1}{n}\frac{\partial}{\partial\mu}\ln W_{\mathrm{MF}} = \beta N$$

and it gives at the saddle point

$$\int_{L_-}^{L_+} dy\sqrt{(L_+^2 - y^2)(y^2 - L_-^2)}\coth\left(\frac{\beta y}{2\sqrt{I}}\right) = 2\pi J^2\sqrt{I}, \tag{A18}$$

where

$$L_\pm^2 = 2\mu \pm 2J, \tag{A19}$$

which is just the condition we[12] found previously for the mean-field quantum spin glass and it gives μ as a function of T and I. For high temperatures, the chemical potential $\mu\to\infty$, while $\mu=\mu_c=J$ at the critical temperature $T_c(I)$ and the critical value I_c is reached when $T_c(I_c)=0$, as shown in the phase diagram in Fig. 1. The high-μ (high-temperature) solution for $Q_\alpha(\omega)$ in Eq. (A17) gives for $\Gamma_0(\vec{k}\omega_1\omega_2)$ in Eq. (A15), when $\mu>J$, the following:

$$\Gamma_0(\vec{k}\omega_1\omega_2) = 1 - (J/\mu)^2 + \frac{I}{2J}(\omega_1^2 + \omega_2^2) + q^2. \tag{A20}$$

Introducing Eq. (A20) into Eq. (A14), rescaling the fields $Q_{\alpha\gamma}\to\frac{1}{\beta JN}Q_{\alpha\gamma}$, and using $g_0(\omega=0,\ \mu=\mu_c)=(\beta J)^{-1}$, we arrive at the effective spin-glass partition functional in the main text. We took explicitly the continuum limit in real space by replacing for vanishing lattice constants

$$\frac{1}{N}\sum_{\vec{k}} \to \int d\vec{k}, \tag{A21}$$

while for finite temperature the sum over Matsubara frequencies $\omega=\frac{2\pi m}{\beta}$ are over the discrete index m. We discuss next the regions with $T>T_c$ and $I<I_c$.

a. Classic paramagnet (high temperature): $\frac{\beta}{\sqrt{I}}\to 0$

In this limit we are in the classical region and the integral in Eq. (A18) can be solved exactly[12] with the result

$$2\left(\frac{\mu}{J}-1\right) = \left(\frac{1}{\sqrt{\beta J}} - \sqrt{\beta J}\right)^2. \tag{A22}$$

b. Quantum paramagnet (low temperature): $\frac{\beta}{\sqrt{I}}\to\infty$

In this limit $\coth\frac{\beta y}{2\sqrt{I}}\approx 1$ and the integral in Eq. (A18) can be solved in terms of elliptic integrals. For $(\frac{\mu}{J}-1)\ll 1$ we obtain

$$2\pi[\sqrt{I_c J} - \sqrt{IJ}] \approx -\frac{4}{3}\left(\frac{\mu}{J}-1\right)\ln\left[\frac{\mu}{J}-1\right]. \tag{A23}$$

Introducing Eq. (A22) into Eq. (A23), we obtain the dotted curve in Fig. 1, which separates the classical region from the quantum paramagnetic region.

3. Integrals in dimensional regularization

In the low-temperature limit the sum over frequencies are replaced by integrals as indicated in Eq. (20). Then we need for $\Gamma^{(2)}$ in Eq. (15) at the critical value $t=0$ (Ref. 19) the following:

$$I_2 = \int d\omega d\vec{p}\frac{1}{p^2 + s^2(\omega_1^2+\omega^2)}\frac{1}{[\vec{p}-\vec{k}]^2 + s^2(\omega^2+\omega_2^2)} = \frac{S_{d+1}}{2s}\Gamma\left(\frac{d+1}{2}\right)\Gamma\left(\frac{3-d}{2}\right) \times\int_0^1 dx\frac{1}{[x(1-x)k^2 + xs^2\omega_1^2 + (1-x)s^2\omega_2^2]^{(3-d)/2}}, \tag{A24}$$

where S_{d+1} is the surface of the unit sphere in $(d+1)$ dimensions. We can see that $\Gamma(\frac{3-d}{2})$ has a dimensional pole at $d_c=5$. Then setting $\epsilon'=5-d$, we obtain the singular contribution in Eq. (21).

To renormalize $\Gamma^{(3)}$ and $\Gamma^{(2,1)}$ in Eqs. (17) and (19), we need to calculate

$$I_3 = \int d\omega d\vec{p} \frac{1}{[p^2 + s^2(\omega_1^2 + \omega^2)]} \frac{1}{[(\vec{p}+\vec{k}_1)^2 + s^2(\omega^2 + \omega_2^2)]} \times \frac{1}{[(\vec{p}+\vec{k}_1+\vec{k}_2)^2 + s^2(\omega^2 + \omega_3^2)]}, \tag{A25}$$

which we do by taking the external momenta and frequencies at the symmetry point[19]

$$k_1^2 = k_2^2 = \omega_i^2 = \kappa^2, \quad \vec{k}_1 \cdot \vec{k}_2 = -\frac{\kappa^2}{2} \tag{A26}$$

and performing the integral in $d+1$ dimensions as in Eq. (24), with the result

$$I_3 = \frac{S_{d+1}}{s} \frac{\Gamma\left(\frac{d+1}{2}\right)\Gamma\left(\frac{5-d}{2}\right)}{\Gamma(3)} \kappa^{-(5-d)} \times \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \frac{1}{[x_1(1-x_1) + x_2(1-x_1-x_2) + s^2]^{(5-d)/2}}. \tag{A27}$$

We see again the dimensional pole in $\Gamma(\frac{5-d}{2})$ at $d_c=5$. Then considering the singular part at the pole in $\epsilon'=5-d$, we obtain the results in Eqs. (22) and (23).

*albath@if.ufrgs.br

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